NOTES ON ASTRONOMICAL OPTICS

Introduction

What follows is a definition of the tentative scope of the subject.

Astronomy is the science dealing with all outside the terrestrial atmosphere.

If one has the good luck to find a just fallen meteor, it can do good astronomy with a pick and shovel.

Some of the best astronomy has been done with complex neutrino detectors.

But the vast majority of astronomical information come as electromagnetic waves, and to extract it is necesary to know the nature of these waves

Next, this knowledge may be applied to the invention, design, construction and use of some astronomical instruments, and to reduction of data given by them.

Historically, most information came from the visible spectrum, and the smallness of the visible wavelength originated the so called geometrical optics, wholly based on the concept of light rays.

The electromagnetic nature of "light" is seen in radiotelescopes, interferometers, polarimeters, diffraction gratings, interference filters, and the detailed analysis of images predicted by geometrical optics.

According to this scheme, the material presented here lack some things, but none is superfluous.

The first section is taken as known from other courses, but it is included here as a quick reference to later topics.

Waves

What is a wave? The simplest example is a sea wave, before breaking.

A graph of the height as a function of time, taken at a fixed pole, is identical to a photograph of the shape, (Fig 1.1). The graph at the pole is obtained by taking $x = \text{constant} = x_0$

The photograph is obtained at a time $t = t_0$. The shape moves as a block at a velocity *c*, that may be positive or negative.

All this can be summarized in the equation

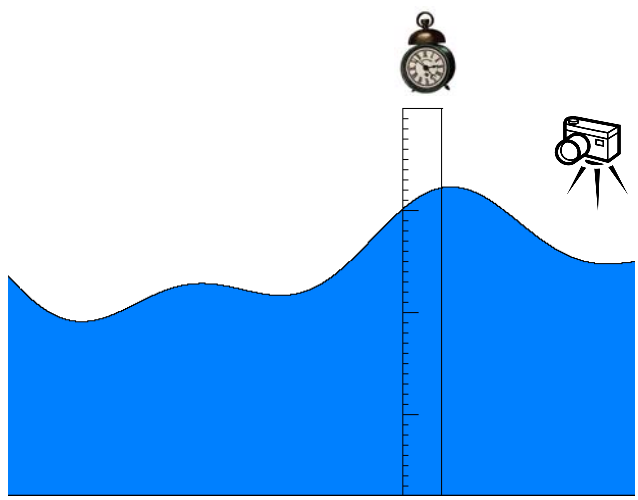


Fig. 1.1

(1.1) $$h = h(x - ct)$$

Where h is the height, x the coordinate and t the time.

For a time $t + \Delta t, h$ has the same value at certain $x + \Delta x,$ such that $\Delta x = c\,\Delta t$

Differential equation of waves

Eq. (1.1) is solution of the differential equation.

(1. 2) $$\frac{\partial^2 h}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 h}{\partial t^2}$$

This equation is called *the wave equation*.

Exercise

Verify it, taking an auxiliar variable $u = x - ct$ and operating on the derivatives

Electric and magnetic units

Brief description of electric and magnetic fields and unit systems.

The MKS system of units must be completed with a new one to include electromagnetic phenomena.

The physical entities are defined from the following experimental laws.

Lorenz law

(1.3) $$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

Where

F = force, q = electric charge, **E** = electric field

v = velocity of q, **B** = magnetic field.

This equation introduces q, **B**, **E**

Coulomb law

(1.4) $$\mathbf{F} = \frac{q_1 q_2}{4\pi\varepsilon_0}\,\frac{\mathbf{r}}{r^3}$$

Where

F = force., q_1, q_2 =charges, **r** = distance from q_1 to q_2. It may also be written as **F** = q_2 $\mathbf{E}_1$; with $\mathbf{E}_1$ =electric field produced by q_1.

This equation is analogous to the law of gravitation, and introduces ε_0 = dimensional constant.

Ampere law

(1.5) $$\oint \mathbf{B} \cdot d\,\mathbf{L} = \mu_0\, i$$

B = magnetic field

d **L**= element of length

i = electric current linked by the circuit of integration.

Definition of electric current: flow of charge = dq / dt

This equation introduces μ_0 = dimensional constant.

Newton's law, **F** = mass x **a**cceleration**,** connects with the mechanical system MKS.

Fundamental units of the MKSQ system

Length: meter = m

Mass: kilogram = k

Time: second = s

Charge: coulomb = q

They are defined by experiments.

Let's see the units of **E**, **B**, q, ε_0, μ_0

The unit of force is the newton, $n = \dfrac{m\,k}{s^2}$

Using (1.3), if v = 0 results in $n = q\,E$; then

(1.6) $$E = \frac{n}{q} = \frac{m\,k}{q\,s^2}$$ This is called "volt / m"

And if E = 0 results in $n = \dfrac{q\,m B}{s}$; then

(1.7) $$B = \frac{n\,s}{q\,m} = \frac{k}{s\,q}$$ This is called "Tesla"

Using (1.4) is $n = \frac{m\,k}{s^2} = \frac{q^2}{\varepsilon_0\,m^2}$; then

(1.8) $$\varepsilon_0 = \frac{q^2 s^2}{m^3 k}$$

Using (1.5) is $B\,m = \frac{k\,m}{s\,q} = \frac{\mu_0 q}{s}$; then

(1.9) $$\mu_0 = \frac{k\,m}{q^2}$$

Units of the product $\varepsilon_0\mu_0$

(1.10) $$\varepsilon_0\mu_0 = \frac{q^2 s^2}{m^3 k}\,\frac{k\,m}{q^2} = \frac{s^2}{m^2} = \frac{1}{velocity^{\,2}}$$

Maxwell equations.

Maxwell equations in vacuum link **E** and **B** without intervention of charges.

(1.11) $$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

(1.12) $$\nabla\times\mathbf{B} = \varepsilon_0\,\mu_0\,\frac{\partial\mathbf{E}}{\partial t}$$

(1.13) $$\nabla\cdot\mathbf{E} = 0$$

(1.14) $$\nabla\cdot\mathbf{B} = 0$$

Equation of electromagnetic waves

Taking $\nabla\times$ in (1.11)

(1.15) $$\nabla\times\left(\nabla\times\mathbf{E}\right) = -\nabla\times\frac{\partial\mathbf{B}}{\partial t}$$

Taking $\frac{\partial}{\partial t}$ in (1.12)

(1.16) $$\nabla\times\frac{\partial\mathbf{B}}{\partial t} = \mu_0\,\varepsilon_0\,\frac{\partial^2\mathbf{E}}{\partial t^2}$$

Using the vectorial identity

(1.17) $$\nabla\times\left(\nabla\times\mathbf{E}\right) = \nabla\left(\nabla\cdot\mathbf{E}\right) - \nabla^2\mathbf{E}$$

The first member of the second term is zero by (1.13)

Results

(1.18) $$\Delta^2\mathbf{E} = \mu_0\varepsilon_0\,\frac{\partial^2\mathbf{E}}{\partial t^2}$$

It is a three dimensional wave equation with $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

It can be verified tha the same equation holds for **B**.

With electromagnetic waves, the magnitude producing observable effects is generally **E**. The field B sustains the propagation.

To think

Electromagnetic waves, luckily for astronomers, needs no material medium to propagate. This property makes them very special and hide facts that Maxwell never imagined

Types of waves

Harmonic waves.

They has the expression

(1.19) $$\mathbf{E} = \mathbf{E}_0 \; sin(kx - \omega t),$$

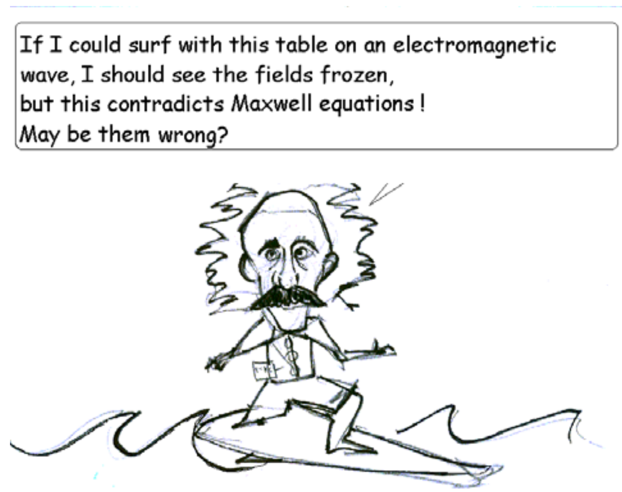
or also

(1.20) $$\mathbf{E} = \mathbf{E}_0 \, e^{i(kx - \omega t)}, \text{ (real part)}$$

The last expression is more manageable mathematically.

Here, k = wave number, ω = angular frequency or pulsation.

$\mathbf{E}_0$ is a complex amplitude and may be written in detail as $\mathbf{E}_0 e^{i\varphi}$. This form will be most used in the discussions.



Alternative notation

(1.21) $$\mathbf{E} = \mathbf{E}_0 \, e^{2\pi i \left(\frac{x}{\lambda} - \frac{t}{\tau} \right)}$$

Here, λ = wavelength, τ = period.

Harmonic plane waves in three dimensions

The former expression say that E is constant in planes normal to the x axis, and the waves travel along x. If they travel in any direction, it turns to be

(1.22) $$\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} \qquad \mathbf{r} = x, y, z \text{ and } \mathbf{k} = k_x, k_y, k_z$$

E is constant in the planes $\mathbf{k}\cdot\mathbf{r}$ = cte, advancing in direction of the wave vector **k** with velocity

(1.23) $$c = \frac{\omega}{k} = \frac{\omega}{\sqrt{k_x^2 + k_y^2 + k_z^2}}$$

Spherical harmonic waves.

They has the expression

(1.24) $$\mathbf{E} = \mathbf{E}_0 \, \frac{1}{r} \, e^{i(kr - \omega t)}$$

They are three dimensional waves, and the factor $1/r$ is required. No spherical waves (harmonic or not), can exist with an equation of the type **E** = **E**(r - c t).

A reason for the factor $1/r$ will be shown later.

Relations among **E, B,** and **k** for a plane harmonic wave

Let $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$

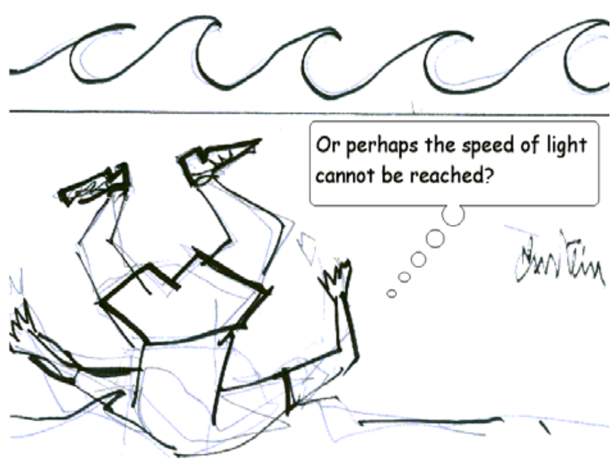
Then

(1.25) $$\frac{\partial \mathbf{E}}{\partial t} = -i\omega \mathbf{E} \;\; ; \;\; \frac{\partial \mathbf{E}}{\partial \mathbf{x}} = i k_x \mathbf{E}$$

ídem for y, z

As operators

(1.26) $$\frac{\partial}{\partial t} \to -i\,\omega \qquad\qquad \nabla \to i\,\mathrm{k}$$



Maxwell equations in this case take the form

(1.27) $\mathbf{k} \times \mathbf{E} = \omega\, \mathbf{B}$

(1.28) $\mathbf{k} \times \mathbf{B} = -\varepsilon_0\, \mu_0\, \omega\, \mathbf{E}$

(1.29) $\mathbf{k} \,.\, \mathbf{E} = 0$

(1.30) $\mathbf{k} \,.\, \mathbf{B} = 0$

This implies that **E, B, k** form an orthogonal thriedron. That is, the wave is transverse.

Radiation pressure

If an electromagnetic wave incides on an electron – as those that compound any piece of matter --, the **E** field moves it in a transverse direction.

By Lorenz law, the associated field B produces a force in direction of **k**. This force has always the same direction, unlike the electrical one that put it into oscillation, so in this case the observable effect is due to B.

Among other things, it produces the comet tails and and keep the stars away from collapse.

Energy flow

The vector

(1.31) $$\mathbf{S} = \frac{\mathbf{E} \times \mathbf{B}}{\mu_0}$$

points in direction **k** and has units

(1.32) $$\frac{m\,k}{s^2 q}\;\frac{k}{s\,q}\;\frac{q^2}{m\,k} = \frac{k}{s^3}$$

The flow density has the same units

(1.33) $$\frac{energy}{s\,m^2} = \frac{k\,m^2}{s^2}\;\frac{1}{s\,m^2} = \frac{k}{s^3}$$

This means that, unless a numerical factor, the energy density carried by an electromagnetic wave is given by **S**. Even though this is not a proof, a complete analysis yields the same result, that is, the numerical factor is 1.

S is called Poynting vector and also, by definition, is the intensity of the beam.

According to (1.23) and (1.27) is $B = k\;E/\omega = E/c = \sqrt{\varepsilon_0 \mu_0}\;E$

Hence the intensity is proportional to E^2

(1.34) $$I = \sqrt{\frac{\varepsilon_0}{\mu_0}}\;E^2$$

Spherical waves has intensity varying with $1/r^2$, in this way energy is conserved because the total flow through any sphere is constant. This is the "reason" for $1/r$ in spherical waves. The true reason is mathematical, but physics thanks it.

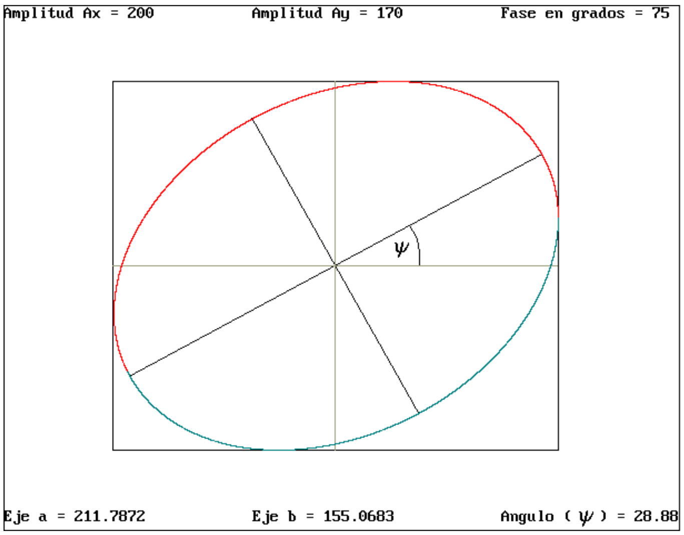


Fig. 1.2

Polarization

Solving for the components of **E** of a plane monochromatic wave in a cartesian system with **k** on the z axis,

(1.35) $E_x = A_x\; cos\;\omega t$

(1.36) $E_y = A_y\; cos(\omega t + \varphi)$

It is easy to verify that if $\varphi = 0$, the **E** vector draws a straight line, an if

$A_y = A_x$ and $\varphi = \pm$ 90 deg, it draws a circle in one or other direction.

In the general case it draws an ellipse with semiaxes *a* and *b*, Fig. 1.2

The ellipse has its major axis tilted an angle ψ from the *x* axis. It is assumed that always is $A_x \geq A_y$

Other cases can be reduced to this one.

The determination of a, b, ψ is acomplex exercise of analytical geometry and will be stated without proof.

Input: A_x, A_y, φ Output: a, b, ψ

Taking

(1.37) $$\tan\,\alpha = \frac{A_y}{A_x}\,,$$ is

(1.38) $$\tan(2\psi) = \tan(2\alpha)\cos\varphi$$

The principal semiaxes *a* y *b* obeys

1.39) $$a^2 + b^2 = A_x^2 + A_y^2$$

Taking

(1.40) $$\sin(2\chi) = \sin(2\alpha)\sin\,\varphi\,,$$

there results

(1.41) $$\frac{b}{a} = \pm\,\tan\chi$$

Exercise

Write a program to generate these figures. (Program "ELIPOLA").

Both versions .BAS and .EXE are in the page of contents.

It may be a matter of discussion the pros and cons of writing a program instead of deriving the formulas. The best would be to make both things. Here there are intended simple and improvised programs because they are the natural evolution of the simple act of using a slide rule, then a pocket calculator, and later a computer. The examples where done with BASIC QB45 , which allows easy graphic programming.

Stokes parameters

Astronomical observations dealing with polarized light generally refer to the so called Stokes parameters. They will be defined here for reference.

	Parameters	Relations
(1.42)	$S_0 = A_x^2 + A_y^2$	$S_0^2 = I^2 = S_1^2 + S_2^2 + S_3^2$
	$S_1 = A_x^2 - A_y^2$	$S_1 = S_0\cos 2\chi\cos 2\psi$
	$S_2 = 2A_xA_y\cos\varphi$	$S_2 = S_0\cos 2\chi\,\sin 2\psi$
	$S_3 = 2A_xA_y\,\sin\,\varphi$	$S_3 = S_0\,\sin 2\chi$

Dielectric refraction and reflection.

Refractive index.

In a dielectric, Maxwell equations remain valid if ε_0 is replaced by ε and μ_0 by μ. ε and μ depends on the material, but in dielectrics it also happens that they are very weakly magnetic, so that it may be set approximately $\mu = \mu_0$. The velocity of propagation in the medium is

(1.43) $$v = \frac{1}{\sqrt{\varepsilon\mu}} \cong \frac{1}{\sqrt{\varepsilon\mu_0}}.$$

The quotient

(1.44) $$n = \frac{c}{v} \cong \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

is the refractive index.

Relation between **B** and **E**.

The relation similar to (1.28)

(1.45) $\mathbf{k}\times\mathbf{B} = -\varepsilon\mu\omega\,\mathbf{E}$, may be put as

(1.46) $kB_{\perp} = -\varepsilon\mu\omega\mathbf{E}_{||}$.

The lower indices $\perp$ (perpendicular) and II (paralell), may refer to the plane of the paper, or more generally to the plane containing **k** and the normal, called plane of incidence.

That is

(1.47) $B_{\perp} = -\sqrt{\varepsilon\mu}\ E_{||} = -\frac{1}{v} E_{||} = -\frac{n}{c} E_{||}$

If a wave has $E_{||} = 0$, it is called tranverse electric (TE), and if $B_{||} = 0$, transverse magnetic (TM). Any wave may be expressed as a linear combination of both.

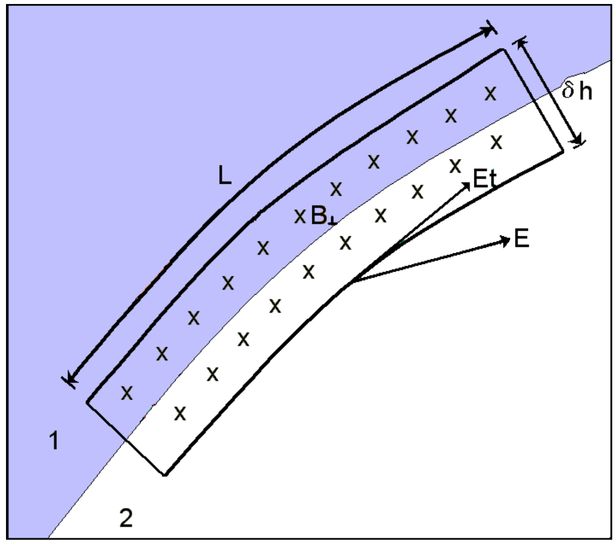


Fig 1.3

Boundary conditions.

In Fig. 1.3 there is shown a boundary between two dielectrics and a path of integration having a part paralell to the boundary of length *L* , then a part normal to it of length *δh* and other two similar closing the circuit. Applying Stokes theorem to the first Maxwell equation results

(1.48) $\oint \mathbf{E}\cdot d\mathbf{L} = -\iint \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$

The first integral refer to the circuit and the second to the surface limited by it

Taking $\delta h << L$ and then $L \to 0$,

(1.49) $\left(E_{1,t} - E_{2,t}\right)L \approx -\frac{\partial B_{\perp}}{\partial t} L\ \delta h \to 0$.

Here the sub-index *t* means tangential, that is, paralell to *L*, and $\perp$ indicates normal to the plane of the circuit.

The expression vanishes because the second member is an infinitesimal of higher order than the first.

The conclusion is $E_{2,t} = E_{1,t}$, i.e., the tangential component of **E** is continuous across the boundary.

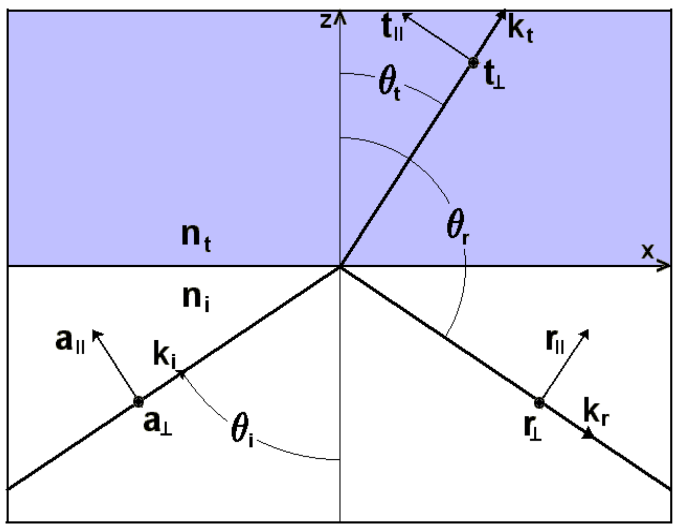


Fig 1.4

The same happens with the tangential component of **B,** by the other equation.

The existence of reflected (r), and transmitted (*t*) waves, and their amplitudes follow from the fulfillment of these boundary conditions.

The directions of the waves follows from the fact that at the boundary the three waves are the same.

In Fig 1.4 a plane wave, with its electric field of complex amplitude **a** and propagation vector $\mathbf{k}_i$ contained in the plane *x,z,* incides on the boundary of two dielectrics lying in the plane *x,y*.

The angles are measured from the z axis.

Directions

At the boundary, the arguments of the three waves *i, r, t* are identical.

(1.50) $\mathbf{k}_i\cdot\mathbf{r} = \mathbf{k}_r\cdot\mathbf{r} = \mathbf{k}_t\cdot\mathbf{r}$

But also, for the three waves is true that

$$\mathbf{k}\cdot\mathbf{r} = k_x x + k_y y + k_z z$$

(1.51) $$= k_x\, x = k\, x \sin\theta$$

Because at the boundary is $z = 0$, and $k_y = 0$ by definition.

The equalities are true for all x

Then

(1.52) $$k_i \sin\theta_i = k_r \sin\theta_r = k_t \sin\theta_t$$

Since the velocity in the medium is $v = \frac{c}{n} = \frac{\omega}{k}$, is $k = \frac{n\omega}{c}$.

Then

(1.53) $$\sin\theta_i = \sin\theta_r \qquad \text{(Law of reflection)}$$

(1.54) $$n_i \sin\theta_i = n_t \sin\theta_t \qquad \text{(Snell law)}$$

In the law of reflection, note that the component z of the reflected vector is negative.

(1.55) $$k_{z,r} = k_r \cos\theta_r = -k_i \cos\theta_i$$

Then

(1.56) $$\cos\theta_i = -\cos\theta_r$$

Amplitudes

Fresnel formulae

The components of the field **E** in the incident wave are

(1.57) $$E_{x,i} = -a_{||} \cos\theta_i \qquad E_{y,i} = a_{\perp}$$

The corresponding of the field **B** are, by (1.47)

(1.58) $$B_{x,i} = -\frac{n_i}{c} a_{\perp} \cos\theta_i \qquad B_{y,i} = -\frac{n_i}{c} a_{||}$$

In a similar way, with the transmitted and reflected waves

(1.59) $$E_{x,t} = -t_{||} \cos\theta_t \qquad E_{y,t} = t_{\perp}$$

(1.60) $$B_{x,t} = -\frac{n_t}{c} t_{\perp} \cos\theta_t \qquad B_{y,t} = -\frac{n_t}{c} t_{||}$$

(1.61) $$E_{x,r} = -r_{||} \cos\theta_r \qquad E_{y,r} = r_{\perp}$$

(1.62) $$B_{x,r} = -\frac{n_i}{c} r_{\perp} \cos\theta_r \qquad B_{y,r} = -\frac{n_i}{c} r_{||}$$

The continuity of the tangential fields is satisfied if

(1.63) $$E_{x,i} + E_{x,r} = E_{x,t}$$

(1.64) $$E_{y,i} + E_{y,r} = E_{y,t}$$

(1.65) $$B_{x,i} + B_{x,r} = B_{x,t}$$

(1.66) $$B_{y,i} + B_{y,r} = B_{y,t}$$

Recalling that $\cos\theta_r = -\cos\theta_i$, the former yields, respectively

(1.67) $$-a_{||} \cos\theta_i + r_{||} \cos\theta_i = -t_{||} \cos\theta_t$$

That is

(1.68) $$\left(a_{||} - r_{||}\right)\cos\theta_i = t_{||} \cos\theta_t$$

(1.69) $$a_{\perp} + r_{\perp} = t_{\perp}$$

(1.70) $$-n_i a_{\perp} \cos\theta_i + n_i r_{\perp} \cos\theta_i = -n_t t_{\perp} \cos\theta_t$$

That is

(1.71) $$\left(a_{\perp} - r_{\perp}\right) n_i \cos\theta_i = n_t t_{\perp} \cos\theta_t$$

(1.72) $$n_i\left(a_{||} + r_{||}\right) = n_t t_{||}$$

Eqs (1.68) y (1.72) form a system of two equations with two unknowns.

Their solutions are $\frac{t_{||}}{a_{||}}$ y $\frac{r_{||}}{a_{||}}$, and eqs (1.69) and (1.71) gives $\frac{t_{\perp}}{a_{\perp}}$ and $\frac{r_{\perp}}{a_{\perp}}$

In order to simplify the notation we set $a_{\perp} = a_{||} = 1$, as they are arbitrary.

Hence the solutions are the Fresnel formulae

(1.73) $$t_{||} = \frac{2 n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

(1.74) $$t_{\perp} = \frac{2 n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

(1.75) $$r_{||} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

(1.76) $$r_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

Using Snell law it may be verified the following alternative expression where the refactive indices has been eliminated

(1.77) $$t_{||} = \frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

(1.78) $$t_{\perp} = \frac{2 \sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

(1.79) $$r_{||} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

(1.80) $$r_{\perp} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

Transmissivity T and reflectivity R

The intensity, or energy flow density, is referred to an area normal to the propagation, and is

(1.81) $$I = \sqrt{\frac{\varepsilon_i}{\mu_0}}\, E^2$$

The density through the boundary is affected by an inclination factor and is

(1.82) $$I_\theta = I \cos\theta$$

The transmitted and reflected intensities are

(1.83) $$T = \sqrt{\frac{\varepsilon_t}{\varepsilon_i}}\, \frac{\cos\theta_t}{\cos\theta_i} |\, t\,|^2 = \frac{n_t \cos\theta_t}{n_i \cos\theta_i} |\, t\,|^2$$

(1.84) $$R = |\, r\,|^2$$

The expression for R simplifies because light is travelling in the same medium

These formulas are valid for each component, ll and $\perp$

Singular angles

Critical angle.

When $n_i > n_t$, there is an interval of angles between

(1.85) $$\theta_i = \theta_{critical} = \arcsin\left(\frac{n_t}{n_i}\right)$$

and 90°, for wich the boundary behaves like a mirror. This is said as total reflection.

Brewster or polarizing angle.

In the alternative expression (1.79) of $r_{||}$, it is evident that if $\theta_i + \theta_t = 90°$ is $r_{||} = 0$, and light is reflected completely polarized. The reflected beam is perpendicular to the transmitted one, and then Snell law yields

(1.86) $$\tan \theta_i = \frac{n_t}{n_i}$$

This particular θ_i will be called θ_B.

Normal incidence.
In this case the formulas are so simplified that is worth to remember them.
The plane of incidence is not defined so there is no polarization.

(1.87) $$r = \frac{n_i - n_t}{n_i + n_t}$$

(1.88) $$t = \frac{2\, n_i}{n_i + n_t}$$

The expression for $r_{||}$ has a negative sign by the way in wich there are defined the vectors in Fig.1.4

Fig 1.5 show $R_{||}$ y $R_{\perp}$ as a function of θ_i. Red pertains to $R_{||}$.

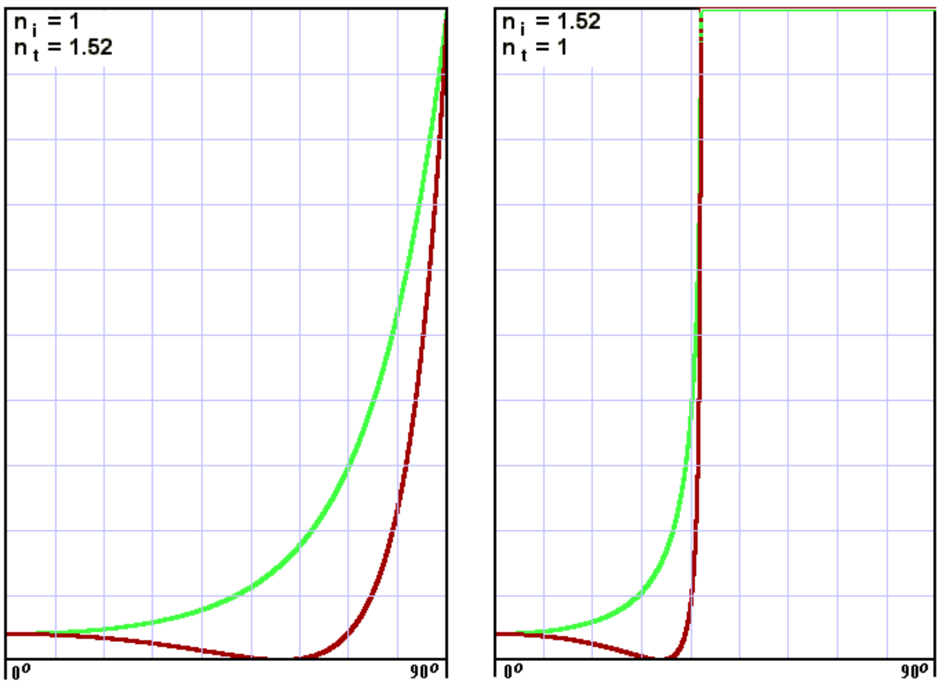


Fig. 1.5

The critical angle is 41.1 degrees and Brewster's is 56.6 ó 33.3 degrees, depending on the case. At normal incidence both components coincide, and the reflectivity is R = 0.042

Exercise
Write a program to generate these figures (Program "FRESNEL").The versions .BAS and .EXE are available in the page of contents
What is the value of $R_{||}$ in Fig 1.4?

Dissipative media
A non dielectric medium (metals, for example), may be characterized by a complex refractive index, $\tilde{n} = n + ik$. Not to be confused this $k =$ index of absorption, with the well known $k =$ modulus of the wave vector $\mathbf{k}$. The name is imposed by the literature.
What is the effect of $\tilde{n}$ oñ the wave?. Let be the plane wave

(1.89) $$E = a\, e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)} = a\, e^{i(k\mathbf{s}\cdot\mathbf{r} - \omega t)}$$

Where $\mathbf{s}$ is a unit vector in the direction of propagation, and $k = \frac{n\omega}{c}$

If now is taken $\tilde{n} = n + ik$, results

$$E = a\, e^{i\left(\frac{\omega}{c}(n+ik)\mathbf{s}\cdot\mathbf{r} - \omega t\right)}$$

(1.90) $$= a\, e^{\frac{-k\omega\mathbf{s}\cdot\mathbf{r}}{c}} e^{i\left(\frac{n\omega\mathbf{s}\cdot\mathbf{r}}{c} - \omega t\right)}$$

It is a wave damped in direction $\mathbf{s}$. The energy is transformed into heat by ohmic dissipation.

Fresnel formulae says that the phase jump is *0* or π if the media are dielectric, but it may be any number if any index is complex.

If there is a complex index, it only makes sense the value of R with n_i real, because the media are infinite and the waves vanish in other case.

An example is the reflectivity of metals, and it is verified an angle similar to Brewster's in wich $R_{||}$ goes through a non-zero minimum.

See example in the annex "program FILMS"